Stochastic Analysis of a Computer System with Priority in Repair Disciplines

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Abstract: **A computer system is analyzed stochastically in detail introducing the aspects of cold standby redundancy and priority in repair disciplines. Initially, one unit (computer system) is operative and the similar unit is kept as spare in cold standby. The hardware (h/w) and software (s/w) fail independently in each unit. A single repair facility is provided with some arrival time. The h/w is repaired at failure while s/w is up-graded as per requirements. Priority is given to the s/w up-gradation over h/w repair. The random variables associated with failure time, repair time, up-gradation time and arrival time are statistically independent. The distributions of h/w and s/w failure times are taken as exponential while the distributions of h/w repair time, s/w up-gradation time and arrival time of the server are assumed as arbitrary. Several reliability characteristics of the system model are obtained using semi-Markov process and regenerative point technique. Graphs for some important measures of system effectiveness are drawn to see effect of various parameters.**

Keywords: **Computer System, Arrival Time, Hardware Repair, Software Up-gradation, Priority and Stochastic Analysis.**

Subject Classification: Primary 90B25 and Secondary 60K110.

1. INTRODUCTION

The incorporation of redundancy is one of the effective techniques to improve the performance and efficiency for the system. Therefore, this technique has been adopted by the researchers including Osaki (1972) and Gupta (1986) to analyze various reliability characteristics of the operating systems. It is commonly assumed that service facility can be made available to the system immediately as and when required. However, this assumption becomes unrealistic when server is already engaged in completion of his pre-assigned jobs. And, so in such a situation, the service facility (server) may be allowed to take some time to arrive at the system. Further, priority may be given in repair disciplines not only to reduce the down time but also to minimize the repair costs. Chander [2005] developed reliability models for a system with priority and arrival time of the server.

In the age of booming technology, computers have brought about a tremendous revolution in every sphere of life and are expected to open more vast fields due to dexterity of those who design and those who develop application programmes. Therefore, scientists and engineers are stressing on the development of reliable h/w and s/w components considering various operational and design polices. Despite of this fact in mind, a little work has been dedicated to the stochastic modelling of computer systems with cold standby redundancy. And, most of the research work has been carried out so far either considering h/w components or s/w components alone.

In view the above and to strengthen the existing literature, here a computer system is analyzed stochastically in detail considering the ideas of redundancy, priority and arrival time of the server. Two identical units of a computer system are taken up - one unit (computer system) is initially operative and the other is kept as spare in cold standby. In each unit, h/w and s/w fail independently from the normal mode. A single repair facility (server) is made available to the system giving some arrival time. Repair of the h/w is done at its failure while s/w is up-graded as per requirements from time to time. Priority is given to the s/w up-gradation over h/w repair. All random variables are statistically independent. The failure time of h/w and s/w is exponentially distributed while the distributions of h/w repair time, s/w up-gradation time and arrival time of the server are taken as arbitrary. The system works as new after h/w repair and s/w up-gradation. The switch over is instantaneous and perfect. The expressions for some measures of system effectiveness such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to h/w repair and software up-gradation, expected number of s/w up-gradations, expected number of visits by the server and profit function are derived in steady state using semi-Markov process and regenerative point technique. Giving particular values to various parameters and costs, the behavior of MTSF, availability and profit have been shown graphically. The applications of the present study can be utilized in the industrial sectors particularly in the hardware and software firms.

2. NOTATIONS

E : The set of regenerative states

O : The unit is operative and in normal mode

Cs : The unit is cold standby

a/b : Probability that the system has hardware / software failure

l1/l2 : Constant hardware / software failure rate

FHUr/FHUR : The unit is failed due to hardware and is under repair / under repair continuously from previous state

FHWr / FHWR : The unit is failed due to hardware and is waiting for repair/waiting for repair continuously from previous state

FSURp/FSURP : The unit is failed due to the software and is under up-gradation / under up-gradation continuously from previous state

FSWRp/FSWRP : The unit is failed due to the software and is waiting for up-gradation / waiting for up-gradation continuously from previous state

w(t) / W(t) : pdf / cdf of arrival time of the server

f(t) / F(t) : pdf / cdf of s/w up-gradation time

g(t) / G(t) : pdf / cdf of h/w repair time

qij (t)/ Qij(t) : pdf / cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]

qij.kr (t)/Qij.kr(t) : pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]

mij : Contribution to mean sojourn time (μi) in state Si when

system transit directly to state Sj so that $\mu_i = \sum_j m_{ij}$ and

mij = $\int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

□/Ó : Symbol for Laplace-Stieltjes convolution/Laplace convolution

~ / * : Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)

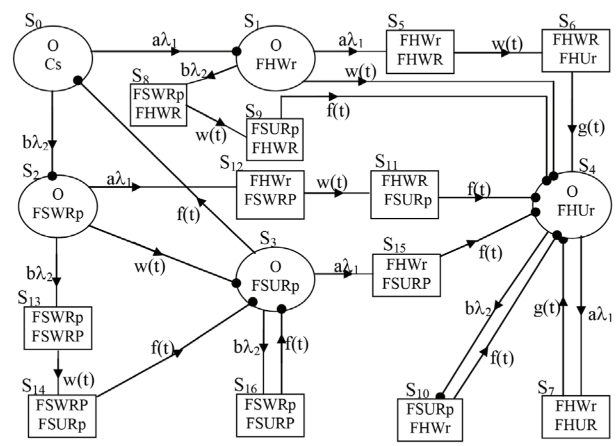

FIG. 1: STATE TRANSITION DIAGRAM

○ Up-state □ Failed state ● Regenerative point

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_j = Q_j(\infty) = \int_0^\infty q_j(t)dt$$ as

$p_{01} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2}$, $p_{02} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2}$, $p_{14} = w^*(a\lambda_1 + b\lambda_2)$,

$p_{18} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2}\left[1 - w^*(a\lambda_1 + b\lambda_2)\right]$, $p_{15} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2}\left[1 - w^*(a\lambda_1 + b\lambda_2)\right]$,

$p_{23} = w^*(a\lambda_1 + b\lambda_2)$, $p_{2,13} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2}\left[1 - w^*(a\lambda_1 + b\lambda_2)\right]$,

$p_{2,12} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-w^*\left(a\lambda_1+b\lambda_2\right)\right]$, $p_{30} = f^*\left(a\lambda_1+b\lambda_2\right)$,

$p_{3,15} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-f^*\left(a\lambda_1+b\lambda_2\right)\right]$, $p_{3,16} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-f^*\left(a\lambda_1+b\lambda_2\right)\right]$,

$p_{40} = g^*\left(a\lambda_1+b\lambda_2\right)$, $p_{47} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\left[1-g^*\left(a\lambda_1+b\lambda_2\right)\right]$,

$p_{4,10} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\left[1-g^*\left(a\lambda_1+b\lambda_2\right)\right]$, $p_{56} = w^*(s)$, $p_{64} = g^*(s)$, $p_{74} = g^*(s)$,

$p_{89} = w^*(s)$, $p_{93} = g^*(s)$, $p_{10,3} = g^*(s)$, $p_{11,4} = f^*(s)$, $p_{12,11} = w^*(s)$, $p_{13,14} = w^*(s)$,

$p_{14,3} = f^*(s)$, $p_{15,4} = f^*(s)$, $p_{16,3} = f^*(s)$, $p_{10,4} = f^*(s)$, (1)

For f (t) = $\theta e^{-\theta t}$, g (t) = $\alpha e^{-\alpha t}$, and $w(t) = \beta e^{-\beta t}$ we have

$p_{13.89} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$, $p_{14.56} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$, $p_{23.13,14} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$,

$p_{24.12,11} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$, $p_{33.16} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\theta}$, $p_{34.15} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\theta}$,

$p_{44.7} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha}$, $p_{43.10} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha}$, (2)

It can be easily verified that

$p_{01}+p_{02} = p_{14}+p_{15}+p_{18} = p_{23}+p_{2,12}+p_{2,13} = p_{30}+p_{3,15}+p_{3,16} = p_{40}+p_{47}+p_{4,10} = p_{14} + p_{14.56} + p_{13.89} = p_{23} + p_{23.13,14} + p_{24.12,11} = p_{30}+p_{33.16}+p_{34.15} = p_{40}+p_{44.7}+p_{43.10} = 1$ (3)

The mean sojourn times (mi) is the state Si are

$\mu_0 = \frac{1}{a\lambda_1+b\lambda_2}$, μ1 =μ2= $\frac{1}{a\lambda_1+b\lambda_2+\beta}$, μ3= $\frac{1}{a\lambda_1+b\lambda_2+\theta}$, μ4 = $\frac{1}{a\lambda_1+b\lambda_2+\alpha}$ (4)

Also

$m_{01}+m_{02} = \mu_0$, $m_{14}+m_{15}+m_{18} = \mu_1$, $m_{23}+m_{2,12}+m_{2,13} = \mu_2$, $m_{30}+m_{3,15}+m_{3,16} = \mu_3$, $m_{40}+m_{47}+m_{4,10} = \mu_4$ (5)

and

$m_{14} + m_{14.56} + m_{13.89} = \mu_1'$, $m_{23} + m_{23.13,14} + m_{24.12,11} = \mu_2'$, $m_{30} + m_{33.16} + m_{34.15} = \mu_3'$,
$m_{40} + m_{44.7} + m_{43.10} = \mu_4'$ (Say)

for f (t) = $\theta e^{-\theta t}$, g(t)= $\alpha e^{-\alpha t}$ and $w(t) = \beta e^{-\beta t}$ (6)

we have

$$\mu_1' = \frac{\alpha(\alpha + a\lambda_1 + b\lambda_2)(\beta+1) + \beta(a\lambda_1 + b\lambda_2)(\beta + b\lambda_2 + a\lambda_1)}{\alpha\beta(a\lambda_1 + b\lambda_2 + \alpha)(\beta + b\lambda_2 + a\lambda_1)}$$

$$\mu_2' = \frac{\theta(\beta + a\lambda_1 + b\lambda_2) + \beta(a\lambda_1 + b\lambda_2)}{\theta\beta(\beta + b\lambda_2 + a\lambda_1)}, \quad \mu_3^1 = \mu_{10}^1 = \frac{1}{\theta}, \quad \mu_4^1 = \frac{\alpha + a\lambda_1}{\alpha(\alpha + a\lambda_1 + b\lambda_2)} \quad (7)$$

4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let ϕ i(t) be the c.d.f. of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ i(t):

$$\phi_i(t) = \sum_j Q_{i,j}(t)(S)\phi_j(t) + \sum_k Q_{i,k}(t)$$

where j is an un-failed regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly. Taking LST of above equation (8) and solving for $\tilde{\phi}_0(s)$

We have

R*(s) = $\dfrac{1 - \tilde{\phi}_0(s)}{s}$

The reliability of the system model can be obtained by taking inverse Laplace transform of equation (9). The mean time to system failure (MTSF) is given by

MTSF = $\lim_{s \to o} \dfrac{1 - \tilde{\phi}_0(s)}{s} = \dfrac{N_1}{D_1}$, where (10)

N1 = $\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{02}p_{23}\mu_3 + p_{01}p_{14}\mu_4$ and D1 = $1 - p_{01}p_{14}p_{40} - p_{02}p_{23}p_{30}$

5. STEADY STATE AVAILABILITY

Let Ai (t) be the probability that the system is in up-state at instant 't' given that the system entered regenerative state Si at t = 0. The recursive relations for Ai (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \quad (11)$$

where j is any successive regenerative state to which the regenerative state Si can transit through n transitions and Mi (t) is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(a\lambda_1 + b\lambda_2)t}, \quad M_1(t) = M_2(t) = e^{-(a\lambda_1 + b\lambda_2)t}\overline{W}(t), \quad M_3(t) = e^{-(a\lambda_1 + b\lambda_2)t}\overline{F}(t), \quad M_4(t) = e^{-(a\lambda_1 + b\lambda_2)t}\overline{G}(t)$$

(12)

Taking LT of above equation (11) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s)=\frac{N_2}{D_2}$$, where

$N_2 = p_{40}(1-p_{33.16})\,\mu_0 + p_{01}p_{40}(1-p_{33.16})\,\mu_1 + p_{01}\,p_{40}\,(1-p_{33.16})\,\mu_2 + p_{02}p_{40}\,(1-p_{24.12,11})\,\mu_3 + [(1-p_{33.16})-p_{02}p_{30}\,(1-p_{24.12,11})]\,\mu_4$

$D_2 = p_{40}(1-p_{33.16})\,\mu_0 + p_{01}p_{40}(1-p_{33.16})\,\mu_1' + p_{01}p_{40}(1-p_{33.16})\,\mu_2' + p_{02}p_{40}\,(1-p_{24.12,11})\,\mu_3' + [(1-p_{33.16})-p_{02}p_{30}\,(1-p_{24.12,11})]\,\mu_4' + p_{4,10}[(1-p_{33.16})-p_{02}p_{30}\,(1-p_{24.12,11})]\,\mu_{10}'$

6. BUSY PERIOD ANALYSIS OF THE SERVER

(a) Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state i at t = 0. The recursive relations $B_i^H(t)$ for are as follows:

$$B_i^H(t) = \mathrm{W_i}^{\mathrm{H}}(\mathrm{t}) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^H(t) \tag{14}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions and WiH (t) be the probability that the server is busy in state Si due to hardware failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4^H(t) = e^{-(a\lambda_1+b\lambda_2)t}\bar{G}(t) + \left[a\lambda_1 e^{-(a\lambda_1+b\lambda_2)} \copyright 1\right]\bar{G}(t) + \left[b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} \copyright 1\right]\bar{G}(t) \tag{15}$$

b) Due to Software up-gradation

Let B_i^S (t)be the probability that the server is busy due to up-gradation of the software at an instant 't' given that the system entered the regenerative state i at t = 0. We have the following recursive relations for B_i^S (t):

$$B_i^S(t) = \mathrm{W_i}^S(\mathrm{t}) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^S(t) \tag{16}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions and W_i^S (t) be the probability that the server is busy in state Si due to up-gradation of the software up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3^S(t) = e^{-(a\lambda_1+b\lambda_2)t}\overline{F}(t) + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2)t} \copyright\ 1)\overline{\mathrm{F}}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2)t} \copyright\ 1)\overline{\mathrm{F}}(t),\quad W_{10}^S(t) = \overline{\mathrm{F}}(t) \tag{17}$$

Taking LT of above equations (14) and (16) and solving for B_0^{*H} (s) and B_0^{*S} (s), the time for which server is busy due to repair and up-gradations respectively is given by

$$B_0^H = \lim_{s \to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2} \quad (18)$$

$$B_0^S = \lim_{s \to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2} \text{, where} \quad (19)$$

$$N_3^H = [p_{34.15} + p_{01}p_{30} + p_{02}p_{30}p_{24.12,11}]\tilde{W}_4^H(0)$$

$$N_3^S = p_{02}p_{40}(1 - p_{24.12,11})\tilde{W}_3^S(0) + [p_{34.15} + p_{01}p_{30} + p_{02}p_{30}p_{24.12,11}]p_{4,10}\tilde{W}_{10}^S(0)$$

and D2 is already mentioned

7. EXPECTED NUMBER OF SOFTWARE UP-GRADATIONS

Let $R_i^S(t)$ be the expected number of up-gradations of the software by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + R_j^S(t)\right] \quad (20)$$

Where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of equation (20) and solving for $\tilde{R}_0^S(s)$. The expected number of software up-gradations per unit time is given by

$$R_0(\infty) = \lim_{s \to 0} s\tilde{R}_0^S(s) = \frac{N_4}{D_2} \text{, where} \quad (21)$$

$N_4 = p_{02}p_{40}[(1 - p_{33,16}) + (1 - p_{23.13,14})] + [p_{34,15} + p_{01}p_{30} + p_{02}p_{30}p_{24.12,11}]p_{4,10}$ and D2 is already mentioned.

8. EXPECTED NUMBER OF VISITS BY THE SERVER

Let Ni (t) be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for Ni(t) are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)(S)\left[\delta_j + N_j(t)\right] \quad (22)$$

where j is any regenerative state to which the given regenerative state i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of equation (22) and solving for $\tilde{N}_0(s)$. The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}$$, where (23)

N5 = p40 (1-p33.16) and D2 is already specified.

9. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

P = $K_0 A_0 - K_1 B_0^H - K_2 B_0^S - K_3 R_0 - K_4 N_0$

where

K0 = Revenue per unit up-time of the system
K1 = Cost per unit time for which server is busy due to hardware repair
K2 = Cost per unit time for which server is busy due to software up-gradation
K3 = Cost per unit up-gradation of the failed software

K4 = Cost per unit visit by the server and $A_0, B_0^H, B_0^S, R_0, N_0$ are already defined.

10. CONCLUSION

For the particular case g (t) = ae^{-at}, $f(t) = \theta e^{-\theta t}$, and $w(t) = \beta e^{-\beta t}$, the results indicate that mean time to system failure (MTSF), availability and profit go on decreasing with the increase of h/w and s/w failure rates (λ1 and λ2) as shown in figures 2, 3 and 4 respectively. It is also revealed that the values of these measures increase with the increase of repair rate (α) and arrival rate (β) of the server. Hence reliability and performance of a computer system having more chances of h/w failure can be improved by the method of redundancy as well as by increasing the h/w repair rate, s/w up-gradation rate and arrival rate of the server.

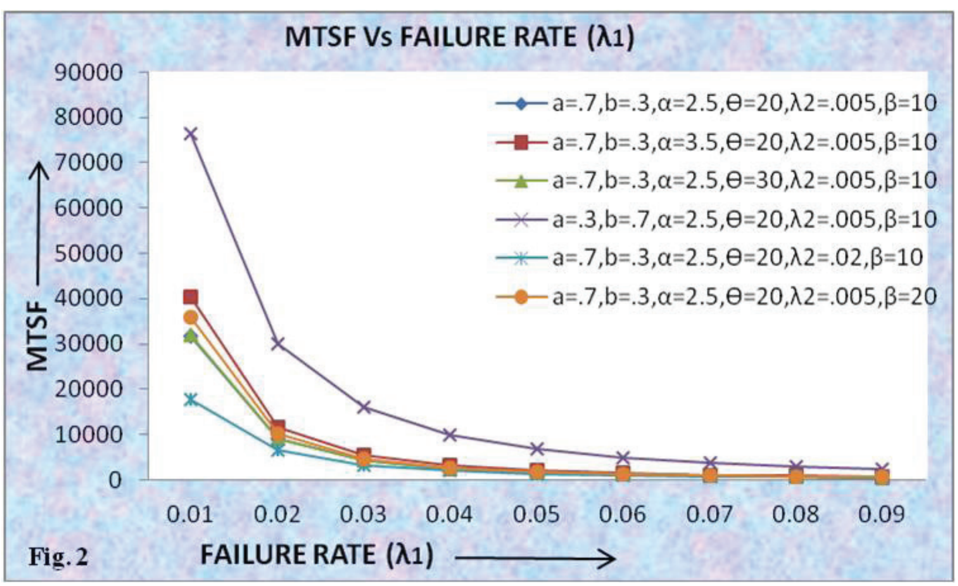

FIG. 2: MSTF VS. FAILURE RATE

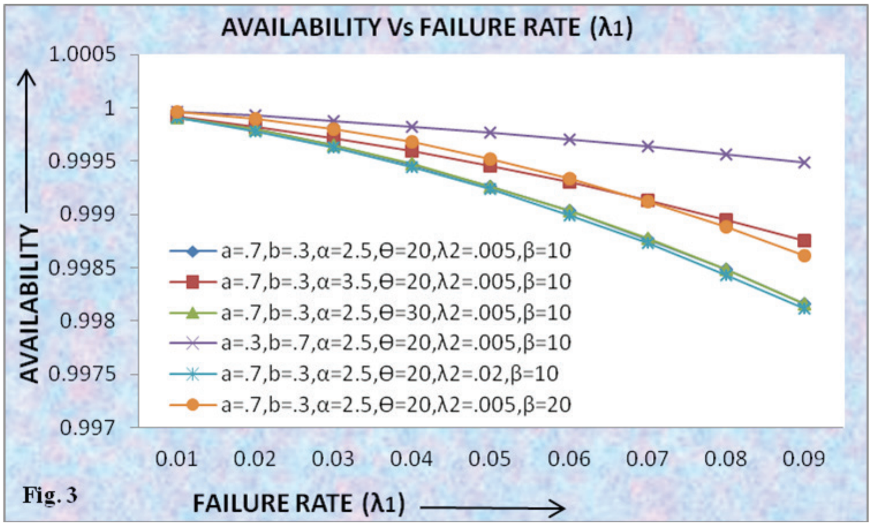

FIG. 3: AVAILABILITY VS. FAILURE RATE

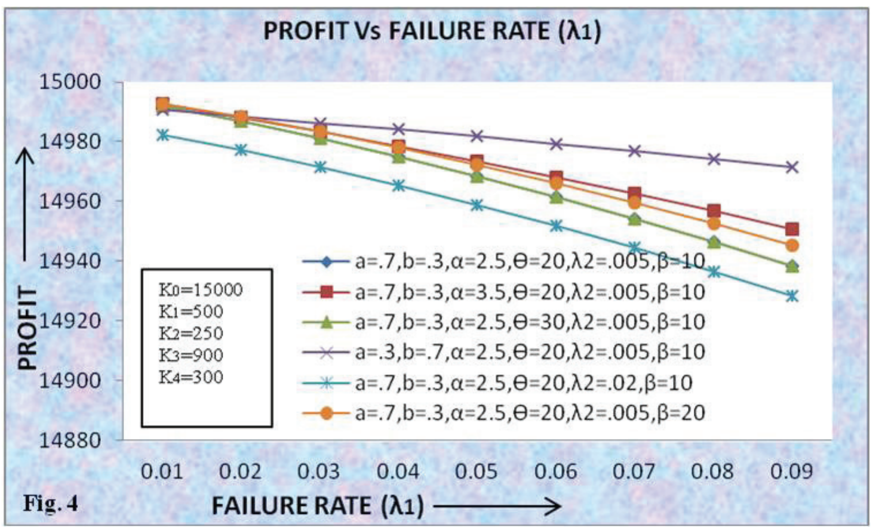

FIG. 4: PROFIT VS. FAILURE RATE

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